Noise Tutorial: Low-frequency CMOS Analog Design

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Outline

- ***Goals***
- Noise Overview
- Resistor Noise Sources
- MOSFET Noise Sources
- Noise of Common Amplifier Topologies
- Examples
- Summary

Goals

- Grasp the importance of noise performance in low-frequency instrumentation design
- Understand the types of noise present in integrated resistors and MOS devices.
- Understand the noise trade-offs of different circuit topologies
- Perform simple noise analysis

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What is noise?

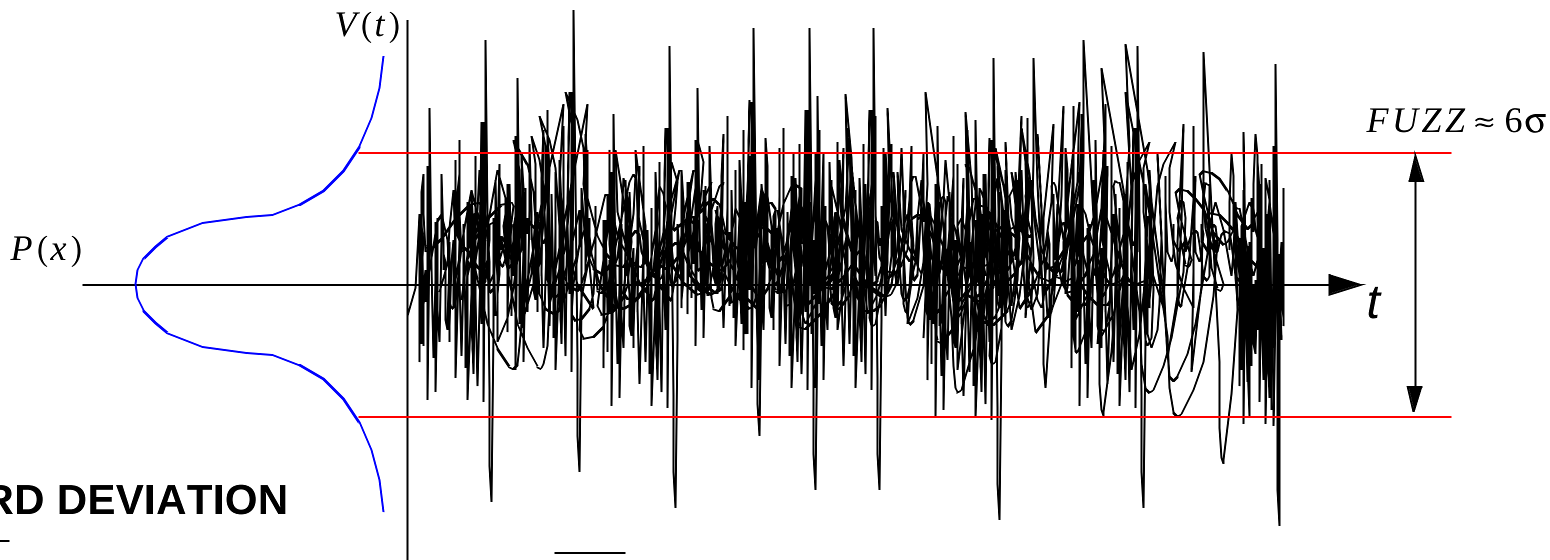
- Unwanted disturbance that interferes with a desired signal
- External: power supply & substrate coupling, crosstalk, EMI, etc.
- Internal: random fluctuations that result from the physics of the devices or materials
- Smallest detectable signal, signal-to-noise ratio (SNR), and dynamic range are determined by noise

$$SNR = \frac{P_{signal}}{P_{noise}} = \frac{V_{rms,\,signal}^2}{V_{rms,\,noise}^2}$$

We will look at internal noise sources and how they affect key performance metrics.

What does it look like?

WHITE NOISE (GAUSSIAN DISTRIBUTION)



MEAN

$$m = 0$$

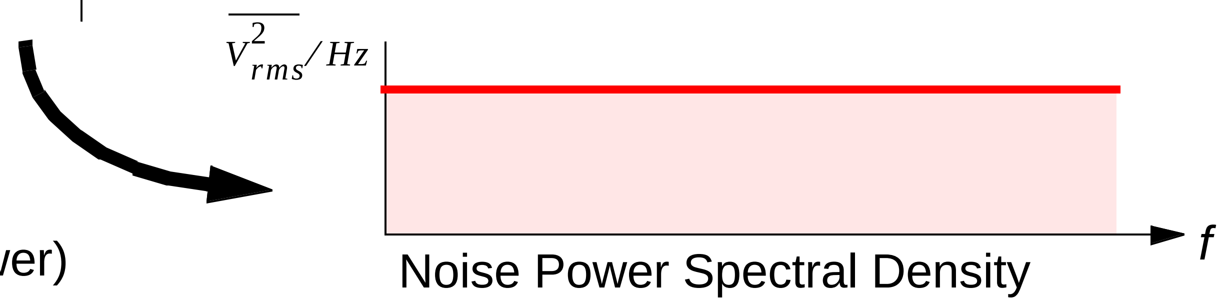
STANDARD DEVIATION

$$\sigma = \overline{V_{rms}}$$

VARIANCE

$$\sigma^2 = \overline{V_{rms}^2}$$

(Normalized Noise Power)

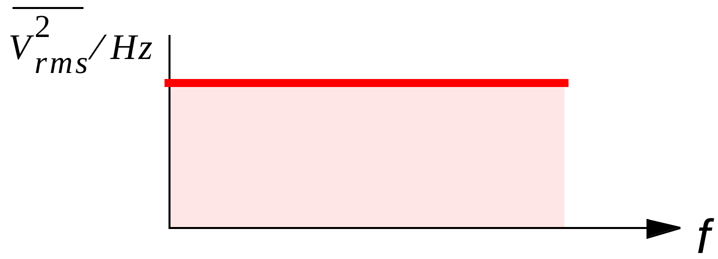


- Random process with zero mean
- Instantateous amplitude is unpredictable
- Average noise power can be measured (variance)

Types of Noise

THERMAL

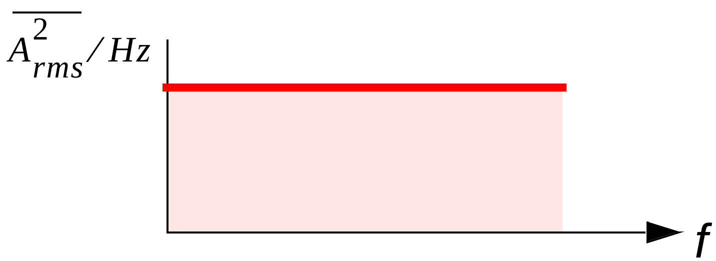
- thermal excitation of charge carriers



$$\frac{\overline{V_{rms}^2}}{\Delta f} = 4kTR \text{ , V}^2\text{/Hz}$$

SHOT

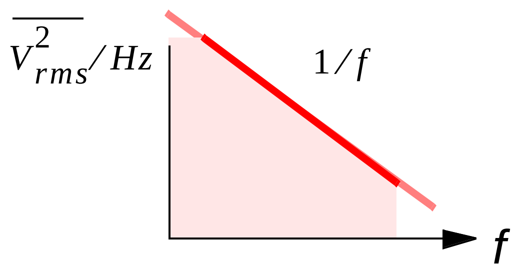
- fluctuations in dc current flow through junctions



$$\frac{\overline{I_{rms}^2}}{\Delta f} = 2qI_{DC} \text{ , A}^2\text{/Hz}$$

FLICKER

- traps in semiconductors



$$\frac{\overline{V_{rms}^2}}{\Delta f} = \frac{K_f \cdot V_{DC}^2}{f} \text{, V}^2\text{/Hz}$$

T = Temperature, K
k = 1.38 x 10^{-23} J/K
q = 1.6 x 10^{-19} C
K_f = Flicker coefficient

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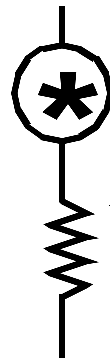
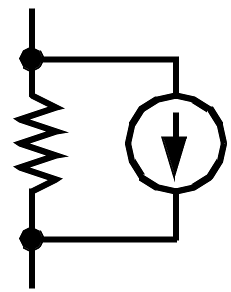
Resistor Thermal Noise





NOISE DENSITY $\dfrac{\overline{V_n^2}}{\Delta f} = 4kTR$, V^2/Hz $\qquad \dfrac{\overline{I_n^2}}{\Delta f} = \dfrac{4kT}{R}$, A^2/Hz

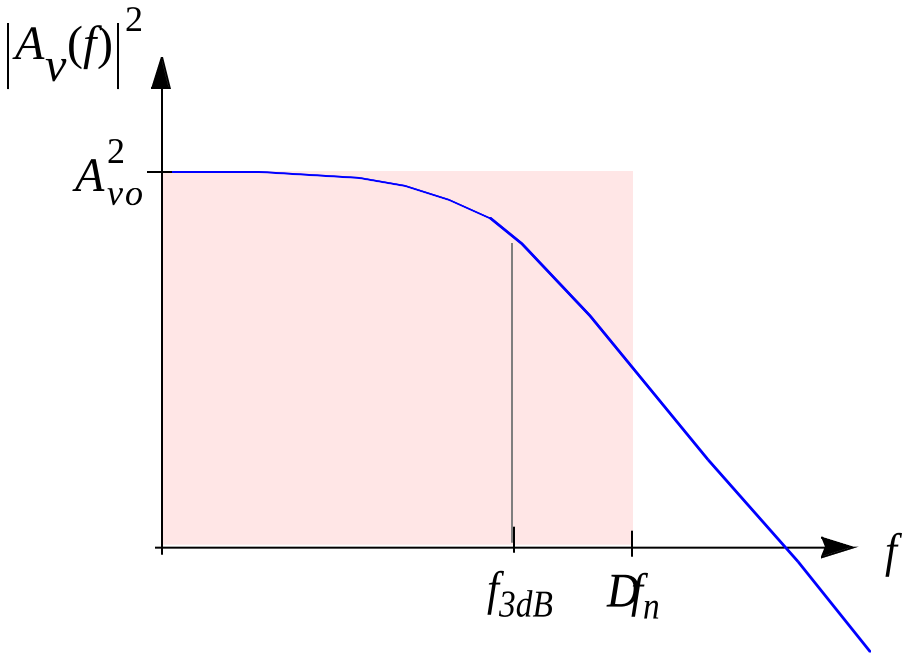
NOISE POWER $\overline{V_n^2} = 4kTR\Delta fn$, V^2 $\qquad \overline{I_n^2} = \dfrac{4kT}{R}\Delta fn$, A^2

Δfn = NOISE BANDWIDTH

- Integrate noise PSD over frequency to arrive at total noise power.
- For white noise, multiply PSD by noise bandwidth.

1kΩ @ 25 ºC
4.05 $nV/\sqrt{Hz}$
4.05 $pA/\sqrt{Hz}$

Noise Bandwidth

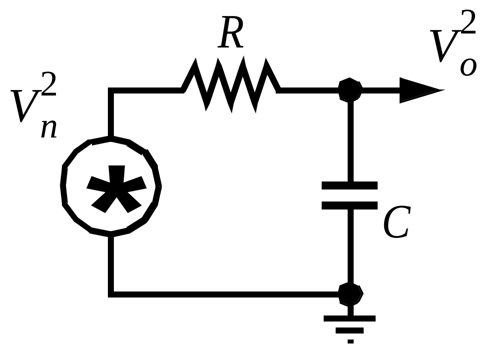


No. Poles	NOISE BW
1	$\Delta f_n = \frac{\pi}{2} \cdot \Delta f_{3dB}$
2	$\Delta f_n = 1.22 \cdot \Delta f_{3dB}$
3	$\Delta f_n = 1.16 \cdot \Delta f_{3dB}$
2nd-order BPF	$\Delta f_n = \frac{\pi}{2} \cdot \Delta f_{3dB}$

$$A_{vo}^2 \cdot \Delta f_n = \int_0^{\infty} |A_v(f)|^2 df$$

- Noise bandwidth is defined for a brickwall transfer function
- Noise bandwidth is not the same as 3dB bandwidth

kT/C Limit



$$\overline{V_o^2} = 4kTR\Delta fn, \text{V}^2$$

$$\overline{V_o^2} = 4kTR\left(\frac{1}{4RC}\right)$$

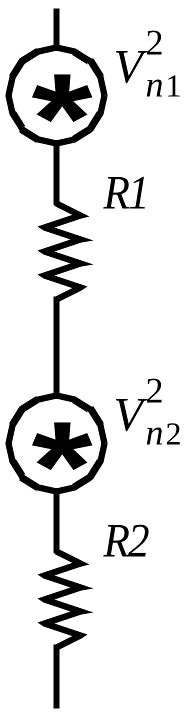
$$\left.\begin{aligned} \Delta f_n &= \frac{\pi}{2} \cdot \Delta f_{3dB} \\ \Delta f_{3dB} &= \frac{1}{2\pi RC} \end{aligned}\right\} \Delta f_n = \frac{1}{4RC}$$

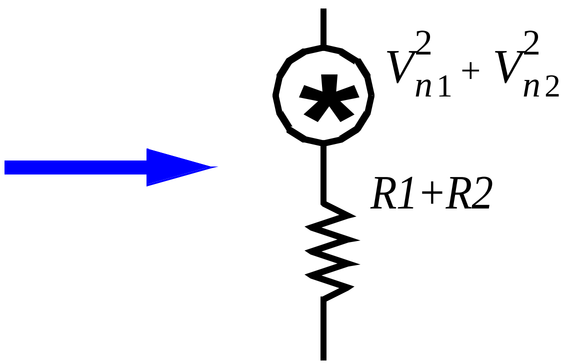
$$\boxed{\overline{V_o^2} = \frac{kT}{C}, \text{V}^2}$$

- Total noise power is independent of R
- Capacitor is noiseless, but accumulates noise from resistor
- This fundamental limit is important for sampled systems

Combining Noise Sources

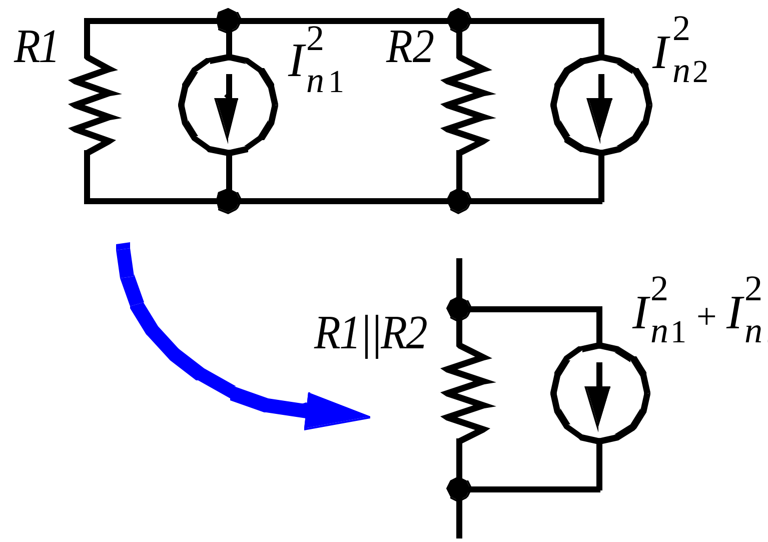
SERIES





$$\overline{V_o^2} = 4kT(R1 + R2)\Delta fn$$

PARALLEL



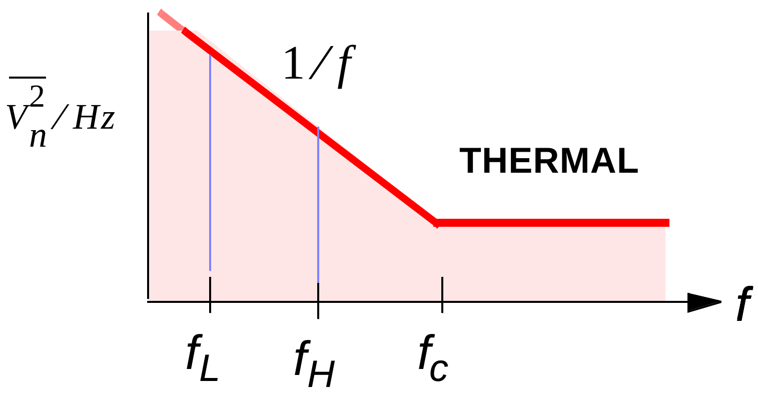
$$\overline{I_o^2} = 4kT\left(\frac{1}{R1} + \frac{1}{R2}\right)\Delta fn$$

- For uncorrelated noise sources, just add the noise powers.
- For correlated sources use:

$$V_o^2 = V_{n1}^2 + V_{n2}^2 + 2cV_{n1}V_{n2}$$

$|c| \leq 1$ **CORRELATION COEFFICIENT**

Resistor Flicker (1/*f*) Noise



$$\frac{\overline{V_{nf}^2}}{\Delta f} = \frac{K_f \cdot V_{DC}^2}{f}, \text{ V}^2/\text{Hz}$$

$$\overline{V_{nf}^2} = \int_{f_L}^{f_H} \frac{K_f \cdot V_{DC}^2}{f} df = K_f \cdot V_{DC}^2 \ln\left(\frac{f_H}{f_L}\right), \text{ V}^2$$

$$\overline{V_{nf}^2} = 2.3 K_f \cdot V_{DC}^2 \quad \text{FOR } f_H/f_L = 10$$

- Constant noise power per decade
- f_c defines 1/*f* corner frequency where flicker noise is equal to thermal noise.

TOTAL RESISTOR NOISE

$$\overline{V_o^2} = 4kTR\Delta fn + K_f \cdot V_{DC}^2 \ln\left(\frac{f_H}{f_L}\right), \text{ V}^2$$

ASSUMES

$\Delta fn \gg f_c$

Noise Index

$$\left.\frac{\overline{V_{nf}^2}}{V_{DC}^2}\right|_{DEC} = \frac{K_f \cdot V_{DC}^2 \ln(10)}{V_{DC}^2} = 2.3K_f$$

$$NI = \sqrt{2.3K_f} \cdot 10^{-6}\text{, mV/V/decade}$$

$$\overline{V_{nf}^2} = \frac{NI^2 \cdot V_{DC}^2}{2.3 \times 10^{-12}} \cdot \frac{1}{f}\text{, V}^2\text{/Hz}$$

$$f_c = \frac{NI^2 \cdot V_{DC}^2}{2.3 \times 10^{-12} \cdot 4kTR}\text{, Hz}$$

- Normalize 1/*f* noise per Volt, per decade of frequency
- Can express 1/*f* noise PSD and f_c in terms of NI

	20*log*(NI)
poly	-15dB
ndiff	-10dB
pdiff	-5dB

[Motchenbacher'93]

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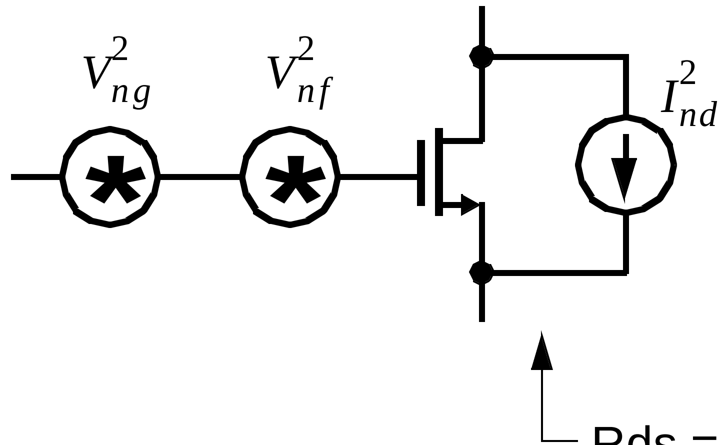
Noise Sources in a MOSFET

DRAIN THERMAL NOISE

$$\frac{\overline{I_{nd}^2}}{\Delta f} = 4kT\gamma g_m \text{, A}^2\text{/Hz}$$

g = { 2/3, long channel; 2-3, short channel

- due to limited channel conductance



Rds = 1/g_m
Long Channel
Saturation

GATE THERMAL NOISE

$$\frac{\overline{V_{ng}^2}}{\Delta f} = 4kTR_G/3 \text{, V}^2\text{/Hz}$$

- can be neglected with good layout

FLICKER NOISE

$$\frac{\overline{V_{nf}^2}}{\Delta f} = \frac{K_f}{WLC_{ox}f} \text{, V}^2\text{/Hz}$$

FOR NMOS

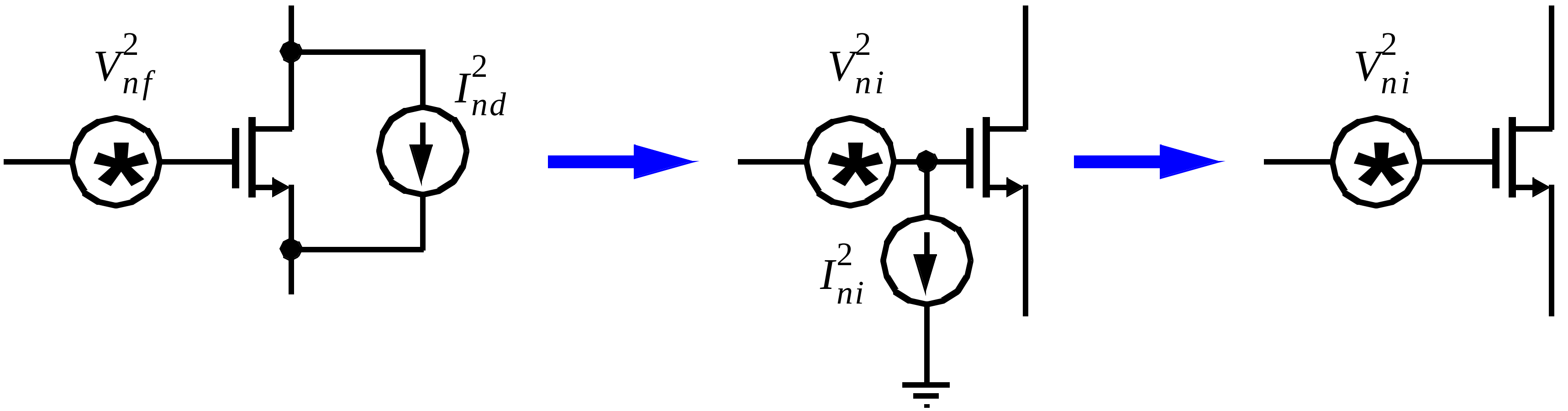
$K_f = 1.2 \times 10^{-24} V^2 \cdot F$

[Razavi'01]

SPICE

$$\frac{\overline{I_{nf}^2}}{\Delta f} = \frac{K_f \cdot I_{DC}^{AF}}{L_{EFF}^2 C_{ox} f} \text{, A}^2\text{/Hz}$$

Input-referred Noise



- Define equivalent input voltage and current noise sources.
- MOSFET input current noise source can be neglected at low frequencies.

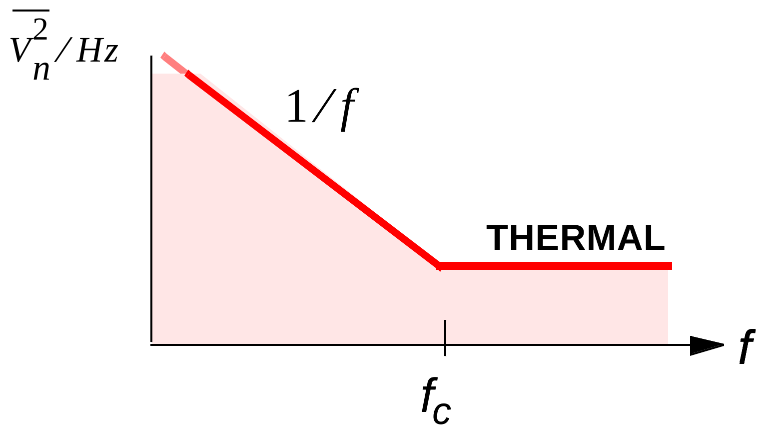
INPUT-REFERRED DRAIN NOISE

$$\frac{\overline{V_{nd}^2}}{\Delta f} = \frac{\overline{I_{nd}^2}}{\Delta f} / g_m^2 = \frac{4kT\gamma}{g_m}, \text{ V}^2/\text{Hz}$$

TOTAL INPUT-REFERRED NOISE

$$\frac{\overline{V_{ni}^2}}{\Delta f} = \frac{4kT\gamma}{g_m} + \frac{K_f}{WLC_{ox}f}, \text{ V}^2/\text{Hz}$$

1/*f* Corner Frequency



$$\frac{4kT\gamma}{g_m} = \frac{K_f}{WLC_{ox}f_c}$$

$$f_c = \frac{K_f \cdot g_m}{4kT\gamma WLC_{ox}}$$

- Defines where flicker noise is equal to thermal noise.
- Need to increase transistor area to reduce flicker noise.

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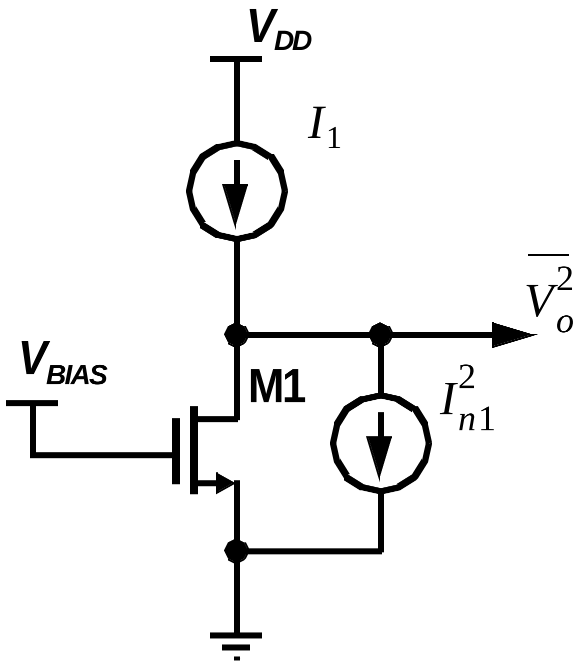
Current Source



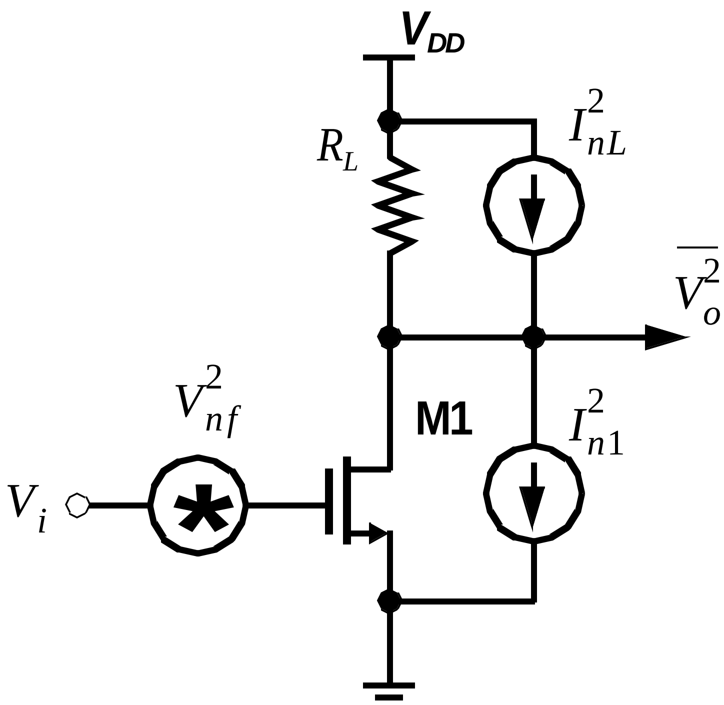
$$\frac{\overline{I_{n1}^2}}{\Delta f} = 4kT\gamma g_m \text{, A}^2\text{/Hz}$$

$$\frac{\overline{V_o^2}}{\Delta f} = \frac{\overline{I_{n1}^2}}{\Delta f} \cdot r_o^2 \text{, V}^2\text{/Hz}$$

$$\frac{\overline{V_o^2}}{\Delta f} = 4kT\gamma g_m r_o^2 \text{, V}^2\text{/Hz}$$

- Neglected flicker noise of M1
- Need to minimize output current noise
- Minimize *gm* for low noise

Common Source



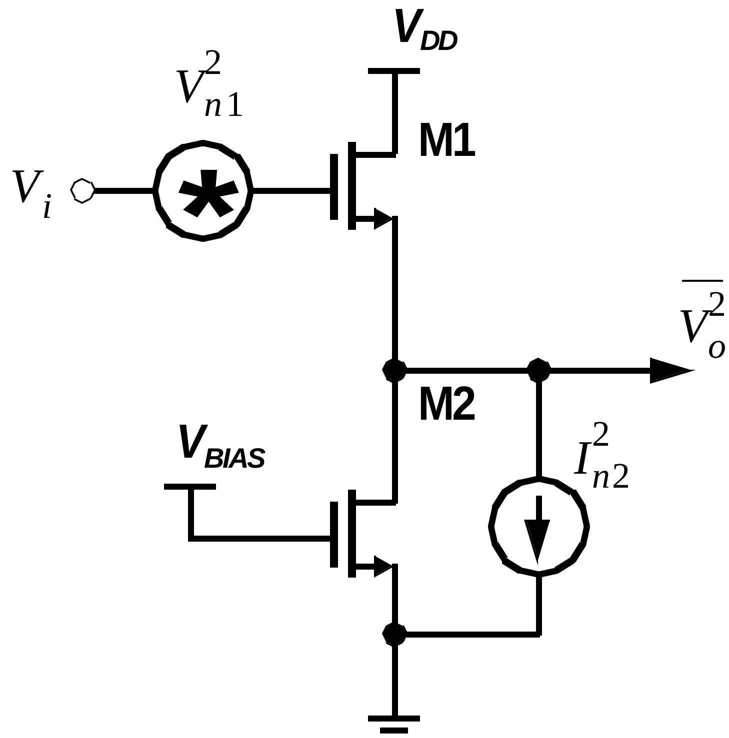
$$\frac{\overline{I_{n1}^2}}{\Delta f} = 4kT\gamma g_m \text{, A}^2\text{/Hz}$$

$$\frac{\overline{I_{nL}^2}}{\Delta f} = \frac{4kT}{R_L} \text{, A}^2\text{/Hz} \qquad \frac{\overline{V_{nf}^2}}{\Delta f} = \frac{K_f}{WLC_{ox}f} \text{, V}^2\text{/Hz}$$

$$\frac{\overline{V_{ni}^2}}{\Delta f} = \frac{4kT\gamma}{g_m} + \frac{K_f}{WLC_{ox}f} + \frac{4kT}{g_m^2 R_L} \text{, V}^2\text{/Hz}$$

- Neglected output conductance of M1
- Maximize g_m , R_L, and transistor area for low noise
- For a differential pair, double the noise

Source Follower



$$\frac{\overline{V_{n1}^2}}{\Delta f} = \frac{4kT\gamma}{g_{m1}} + \frac{K_f}{WLC_{ox}f}, \ \mathrm{V^2/Hz}$$

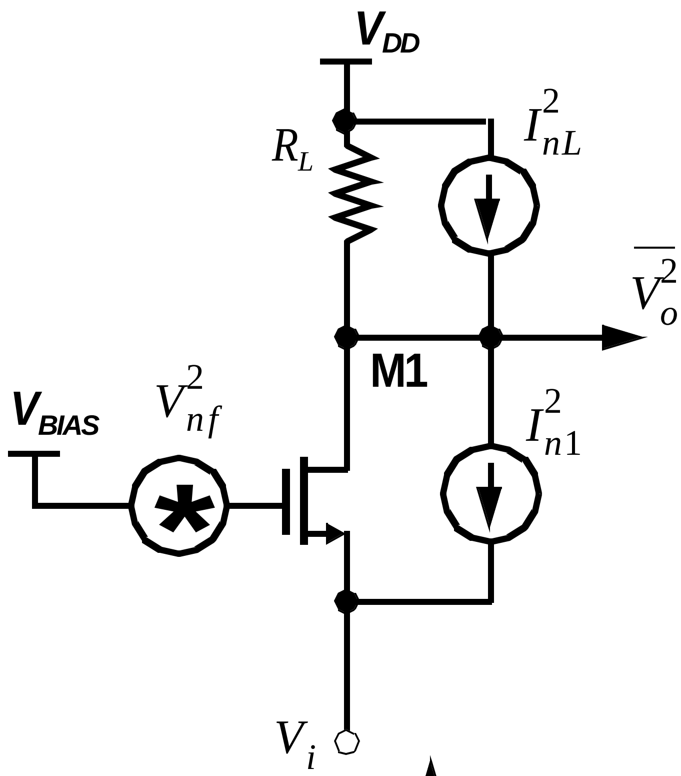
$$\frac{\overline{I_{n2}^2}}{\Delta f} = 4kT\gamma g_m, \ \mathrm{A^2/Hz}$$

$$\frac{\overline{V_{ni}^2}}{\Delta f} = \frac{\overline{V_{n1}^2}}{\Delta f} + \frac{\frac{\overline{I_{n2}^2}}{\Delta f} \cdot \left(\frac{1}{g_{m1}} \parallel r_{o1}^2 \parallel r_{o2}^2\right)^2}{A_v^2}, \ \mathrm{V^2/Hz}$$

WHERE $A_v \approx 1$

- Neglected flicker noise of M2
- VERY NOISY, AVOID

Common Gate



Rs = $1/g_m$
(neglecting
body effect)

$$\frac{\overline{I_{nL}^2}}{\Delta f} = \frac{4kT}{R_L}\text{, A}^2\text{/Hz} \qquad \frac{\overline{V_{nf}^2}}{\Delta f} = \frac{K_f}{WLC_{ox}f}\text{, V}^2\text{/Hz}$$

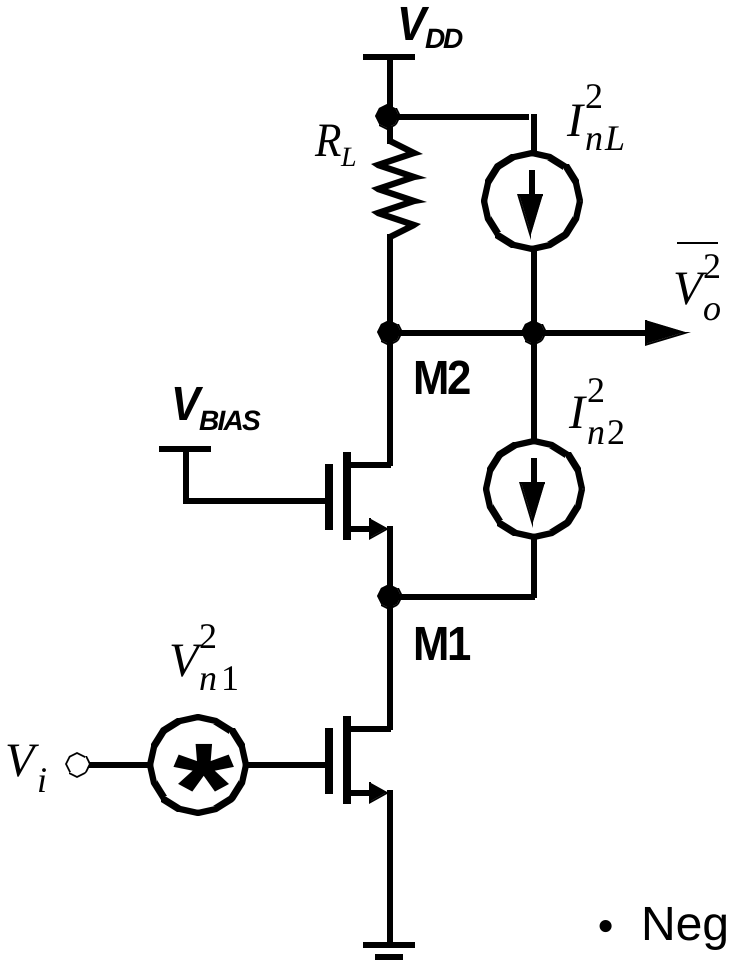
$$\frac{\overline{I_{n1}^2}}{\Delta f} = 4kT\gamma g_m\text{, A}^2\text{/Hz}$$

$$\frac{\overline{V_{ni}^2}}{\Delta f} = \frac{4kT\gamma}{g_m} + \frac{4kT}{g_m^2 R_L} + \frac{K_f}{WLC_{ox}f}\text{, V}^2\text{/Hz}$$

$$\frac{\overline{I_{ni}^2}}{\Delta f} = \frac{4kT}{R_L} + \frac{g_m^2 K_f}{WLC_{ox}f}\text{, A}^2\text{/Hz}$$

- Neglected output conductance of M1
- Define equivalent input voltage and current noise sources
- Input noise current cannot be ignored
- Transfers load current noise to the input

Cascode



$$\frac{\overline{V_{n1}^2}}{\Delta f} = \frac{4kT\gamma}{g_{m1}} + \frac{K_f}{WLC_{ox}f}, \text{ V}^2/\text{Hz}$$

$$\frac{\overline{I_{nL}^2}}{\Delta f} = \frac{4kT}{R_L}, \text{ A}^2/\text{Hz}$$

$$\frac{\overline{V_{ni}^2}}{\Delta f} = \frac{4kT\gamma}{g_{m1}} + \frac{K_f}{WLC_{ox}f} + \frac{4kT}{g_{m1}^2 R_L}, \text{ V}^2/\text{Hz}$$

- Neglected output conductance of M1
- M2's noise contribution is negligible at low frequencies
- Capacitance at M1's drain increase noise at high frequencies

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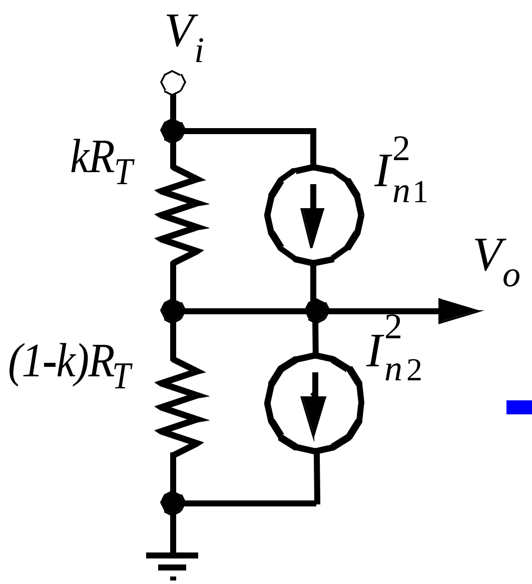
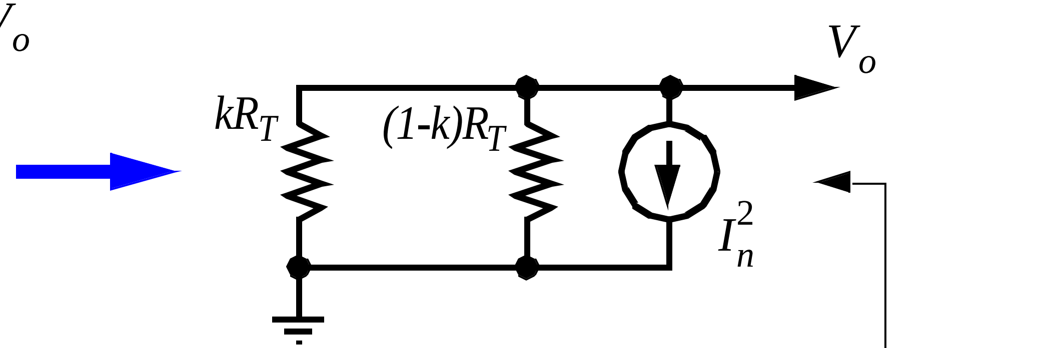
Variable Resistor





WORST CASE

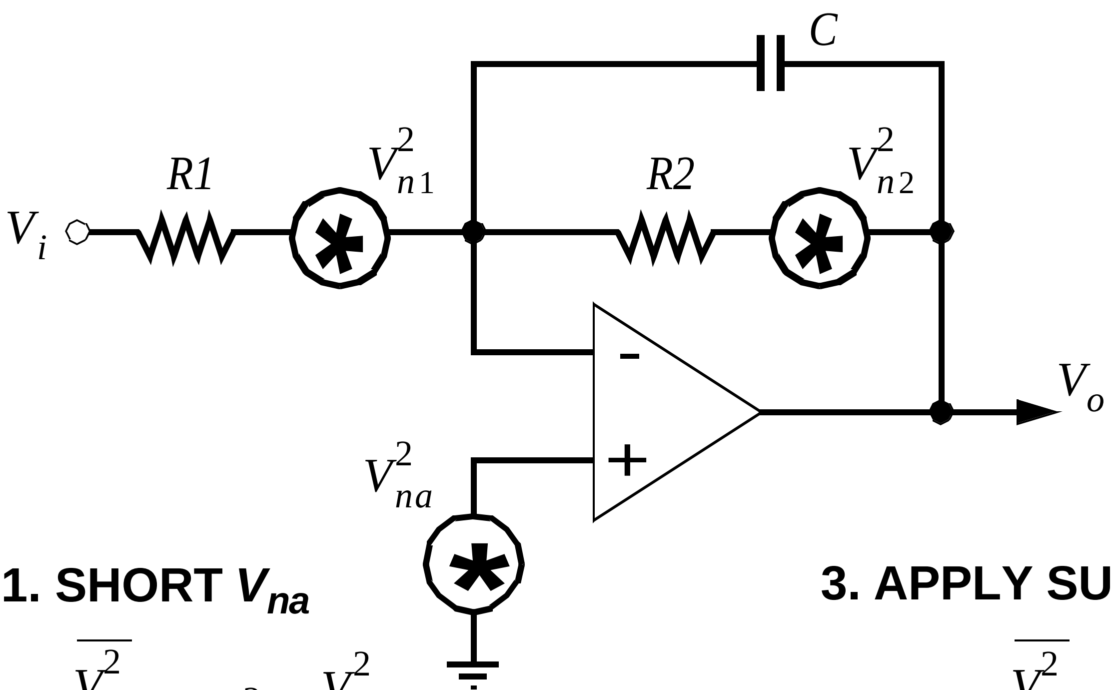
$k = 0.5$

$R_{eq} = R_T/4$

$$\frac{\overline{I_n^2}}{\Delta f} = \frac{4kT}{R_{eq}}, \text{ A}^2/\text{Hz}$$

$$\frac{\overline{I_n^2}}{\Delta f} = \frac{16kT}{R_T}, \text{ A}^2/\text{Hz}$$

Inverting Operational Amplifier Filter



- Negligible input noise current
- Ignore 1/f noise
- Low frequency operation

where

$$A_{vi} = -\frac{R2}{R1} \qquad A_{vni} = 1 + \frac{R2}{R1}$$

$$\Delta f_n = \frac{\pi}{2} \cdot \frac{1}{2\pi R_2 C} = \frac{1}{4 R_2 C}$$

1. SHORT V_{na}

$$\frac{\overline{V_{ni}^2}}{\Delta f} = V_{n1}^2 + \frac{V_{n2}^2}{A_{vi}^2}$$

2. SHORT V_i

$$\frac{\overline{V_{ni}^2}}{\Delta f} = V_{na}^2\left(\frac{A_{vni}^2}{A_{vi}^2}\right) = V_{na}^2\left(1 + \frac{R1}{R2}\right)^2$$

3. APPLY SUPERPOSITION

$$\frac{\overline{V_{ni}^2}}{\Delta f} = V_{n1}^2 + \frac{V_{n2}^2}{A_{vi}^2} + V_{na}^2\left(1 + \frac{R1}{R2}\right)^2$$

FOR LARGE GAIN, R2 >> R1

$$\frac{\overline{V_{ni}^2}}{\Delta f} = V_{n1}^2 + \frac{V_{n2}^2}{A_{vi}^2} + V_{na}^2 \text{, } V^2/\text{Hz}$$

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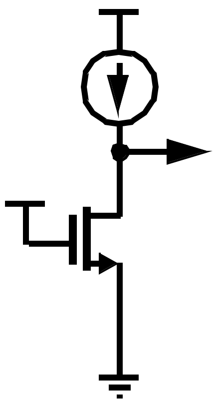
What We Learned



NOISE IN GENERAL

- Thermal noise and flicker noise are the major sources of headaches for low-noise, low frequency instrumentation design in CMOS.
- Noise bandwidth is not the same as 3dB bandwidth.
- Capacitors are noiseless, but they can accumulate noise.
- Resistors can have flicker noise, but only when there is current flow.

NOISE IN MOSFETS

- Need to increase transistor area to reduce flicker noise.
- Current source: Minimize *gm* for low noise current
- Common source: Maximize g_m , R_L, and transistor area for low noise
- Source follower: VERY NOISY, AVOID
- Common gate: Transfers load current noise to the input. Input noise current cannot be ignored.
- Cascode: Cascode transistor noise contribution is negligible at low frequencies.

References

1. P.H. Garret, "Analog I/O Design: Acquisition, Conversion, Recovery," *Reston Publishing*, *Reston*, *VA*, pp. 264, 1981.

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